HYDRODYNAMIC ANALOGIES OF THE PHENOMENA OF IGNITION AND EXTINCTION

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The problem of determining the steady-state dissipative heating of a Newtonian liquid moving in a round tube of finite length, taking account of the dependence of the viscosity on the temperature, is formulated. The possibility of a jumpwise transition from low-temperature flow conditions with small mass flow rates to high-temperature flow conditions with large mass flow rates, and the reverse with a gradual change in the pressure drop, is established. This phenomenon is brought about by hydrodynamic thermal ignition and extinction; an analytical description of it is given.

1. The dissipative evolution of heat with a nonlinear dependence of the viscosity on the temperature can lead to the development of a hydrodynamic thermal explosion. A theoretical justification of this phenomenon for Couette flow, with a time-invariable shear stress at a movable boundary was obtained in [1] and an experimental confirmation in [2]. A hydrodynamic thermal explosion was predicted for the first time for a pressurized type of flow [3]. With the solution of the nonisothermal problem of the flow of a viscous liquid in a tube of infinite length under the action of a given pressure gradient, in [3] as well as in [4-6], there was proved the existence of a critical value of the pressure gradient, above which there is no steady-state solution of the problem. It is necessary to bear in mind a certain idealization of the above statement of the problem, consisting in the consideration of a tube of infinite length, and in the assignment, as a starting parameter, of the pressure gradient whose value is usually sought. In the case of a tube of finite length, for time-invariable boundary conditions, steady-state flow conditions always exist [7].

It is shown in the present article that, if the thermal initial section and the limited nature of the residence time of the liquid in the tube are taken into consideration, then the appearance of nonlinearity of the dissipative function of the heat evolution manifests itself not in the form of a hydrodynamic thermal explosion, i.e., in an essentially nonsteady-state development of the process with progressive self-heating of the liquid but in the possibility of a sharp jumpwise transition, with a continuous change of the pressure drop from low-temperature steady-state flow conditions to high-temperature conditions and back; under these circumstances, the critical conditions for the transitions do not coincide (the hysteresis effect). This phenomenon belongs to the same class as the phenomena of ignition and extinction in the theory of combustion [8, 9] and in magnetic hydrodynamics [10-12] due to the nonlinearity of the dependences, respectively, of the reaction rate and the electrical conductivity on the temperature. The above analogies make it possible to use the "zero-dimensional" method, which has been effectively applied in the theory of combustion [13] for a theoretical description of the phenomena of hydrodynamic ignition and extinction.

Let us consider the steady-state flow conditions of a viscous incompressible liquid in a round cylindrical tube of radius R and length l. The flow is due to the pressure drop at the inlet and outlet of the tube $\Delta p = p(0) - p(l)$.

We make the assumption of thermal flow conditions, considerably simplifying the problem. We shall assume that there is no distribution of the pressure in a cross section of the tube and that the heat transfer along the axis of the tube is not significant.

The first assumption holds with small Biot numbers $Bi = \alpha R/\lambda \ll 1$, and the second with large Peclet numbers $Pe = Q/(\pi R a) \gg 1$ (here α is the heat-transfer coefficient; λ and a are the coefficients of thermal

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conductivity and thermal diffusivity of the liquid; Q is the mass flow rate of the liquid in unit time). If the condition $Bi \ll 1$ is not observed and there is a pressure distribution in the cross section, then we shall use the concepts of the mean temperature and the effective heat-transfer coefficient α in a way similar to that used in the problem of a thermal explosion [9]. With application to the given case, an expression for α is obtained below (section 4).

With the assumptions made, the heat-balance equation, referred to unit volume of the liquid under steady-state conditions, has the form

$$\frac{c\rho}{\pi R^2} Q \frac{dT}{dz} = q(T) - \frac{2\alpha}{R}(T - T_0) \tag{1.1}$$

Here c is the heat capacity; ρ is the density; T_0 is the temperature of the surrounding medium; q(T) is the dissipation function. In the case where there is no dissipation of energy $[q(T) \equiv 0]$, from Eq. (1.1) in [14, 15] a formula is obtained for determining the steady-state heating (cooling) of the liquid moving in the tube. In the general case, to determine the dissipative evolution of heat, together with Eq. (1.1) it is necessary to consider the hydrodynamic equations and the rheological equation of the liquid.

To determine the form of q(T) we adopt the following assumptions with respect to the character of the flow of the liquid:

1) the flow is laminar, one-dimensional, and steady-state, i.e., $v = v_z$ is singular and not equal to zero; the component of the velocity and $\partial v/\partial t = 0$;

2) in the initial cross section of the tube z = 0, the flow of liquid has a definite laminar velocity profile, corresponding to isothermal flow.

The latter assumption is equivalent to the observance of the conditions $l_1 \ll l_2$ and $l_1 \ll l$ (l_1 and l_2 are, respectively, the lengths of the sections of hydrodynamic and thermal stabilization) and is practically always satisfied.

Since the expression for the dissipation function

$$q(T) = \frac{2}{R^2}\int_0^R \sigma D r\, dr \tag{1.2}$$

(the factor $2/R^2$ is a result of averaging) contains the shear stress σ and the deformation rate D, the heat-balance equation (1.1) must be supplemented by the hydrodynamic equations and a rheological equation connecting σ and D. For the flow under analysis we have [15]

$$\sigma = \sigma_{rz} = \frac{dp}{dz}\frac{r}{2}, \qquad D = \frac{dv}{dr} \tag{1.3}$$

where dp/dz is the pressure gradient. Substituting (1.3) into (1.2), we obtain

$$q(T) = \frac{1}{R^2}\int_0^R \frac{dp}{dz}\frac{dv}{dr} r^2\, dr \tag{1.4}$$

Let us consider further the case of a Newtonian liquid

$$\sigma = \mu(T) D \tag{1.5}$$

with a Reynolds dependence of the viscosity on the temperature

$$\mu(T) = \mu_0 e^{-k(T-T_0)} \quad (\mu_0,\ k = \text{const}) \tag{1.6}$$

Taking account of the boundary condition v(R) = 0, we integrate the expression for the volumetric flow rate Q in unit time by parts

$$Q = 2\pi\int_0^R v(r) r\, dr = -\pi\int_0^R \frac{dv}{dr} r^2\, dr \tag{1.7}$$

Using (1.3) and (1.5), we obtain

$$Q = -\frac{\pi}{2}\int_0^R \frac{dp}{dz}\frac{r^3}{\mu(T)}\, dr \tag{1.8}$$

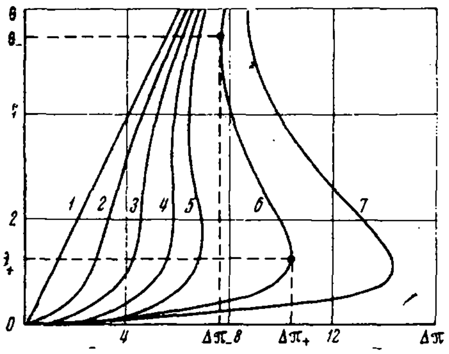

Fig. 1

By virtue of the assumption adopted and from the condition of continuity, the dissipation heating of the moving liquid does not change the velocity profile. Since the viscosity depends on the temperature, and the temperature varies only along the length of the tube, there is a pressure drop along the tube, determined from (1.8) by the relationship

$$\frac{dp}{dz} = -\frac{8\mu_0 Q}{\pi R^4} e^{-k(T-T_0)} \tag{1.9}$$

The relationships for the mass flow rate (1.8) and the pressure gradient (1.9) permit representing the expression for the dissipation function (1.4) in the form

$$q(T) = \frac{8\mu_0 Q^2}{\pi^2 R^6} e^{-k(T-T_0)} \tag{1.10}$$

The heat-balance Eq. (1.1) taking account of (1.10) is represented in the following manner:

$$\frac{c\rho}{\pi R^2} Q \frac{dT}{dz} = \frac{8\mu_0}{\pi^2 R^6} Q^2 e^{-k(T-T_0)} - \frac{2\alpha}{R}(T - T_0) \tag{1.11}$$

Let us consider the case where the temperature of the liquid at the inlet to the tube and the external medium are equal, i.e.

$$z = 0, \quad T = T_0 \tag{1.12}$$

We integrate Eq. (1.9)

$$\Delta p = p(0) - p(l) = \frac{8\mu_0 Q}{\pi R^4} \int_0^l e^{-k(T-T_0)}\, dz \tag{1.13}$$

In what follows we shall distinguish two sets of flow conditions: the mass flow rate of the liquid is given, Q; the pressure drop Δp in a section of the tube with a length l is given. In the first case, to determine the steady-state dissipative heating, it is sufficient to solve the differential Eq. (1.11), and, in the second case, the system of the differential and integral Eqs. (1.11) and (1.13).

We bring Eqs. (1.11) and (1.13) into dimensionless form. We choose the following dimensionless variables and parameters:

$$\omega = \frac{8k\mu_0 l}{c\rho\pi R^4} Q, \quad \Delta\pi = \frac{k}{c\rho}\Delta p, \quad B = \frac{16k\mu_0 l^2}{(c\rho)^2 R^3}\alpha$$
$$\theta = k(T - T_0), \quad \xi = z/l \tag{1.14}$$

Then, the problem under consideration comes down to solution of the equations

$$\omega\frac{d\theta}{d\xi} = \omega^2 e^{-\theta} - B\theta, \quad \Delta\pi = \omega\int_0^1 e^{-\theta}\, d\xi, \quad \xi = 0, \quad \theta = 0 \tag{1.15}$$

In what follows we shall leave as dimensionless variables the designations of the dimensional quantities defining their physical meaning, i.e., ω the mass flow rate; $\Delta\pi$ the pressure drop; θ the temperature; ξ the longitudinal coordinates; B the heat-transfer coefficient.

2. Let us examine the results of a numerical solution of the problem (1.15). The dependence of the steady-state value of the temperature at the outlet from the tube $\theta(1) = \theta_1$ on the pressure drop $\Delta\pi$ is of interest. Figure 1 shows a family of such curves, 1-7, corresponding to values of the heat-transfer coefficient B = 0, 10, 30, 60, 100, 250, and 500.

With B = 0 (straight line 1), which corresponds to adiabatic flow, we have $\theta_1 = \theta_1^\circ = \Delta\pi$. This result can be obtained from Eqs. (1.5). As is shown in [7], for an arbitrary value of B with $\omega \to \infty$

$$\theta_1 \approx \theta_1^\circ = \Delta\pi, \quad \Delta\pi \approx \ln\omega \tag{2.1}$$

This means that the adiabatic straight line is an asymptote for all the temperature curves.

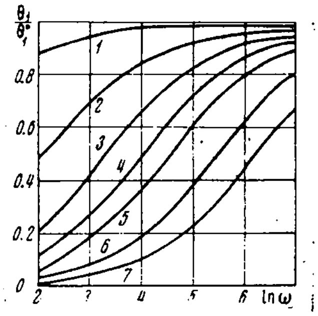

Fig. 2

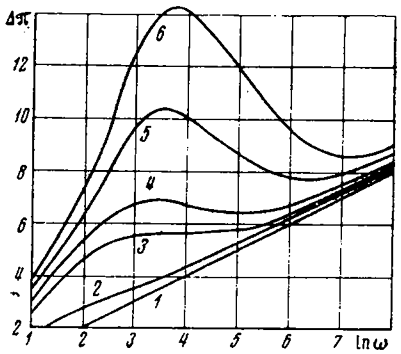

Fig. 3

With sufficiently small values of B = 10, 30 (curves 2, 3), the temperature θ_1 rises monotonically and only one set of steady-state conditions is possible. The transition along the temperature curves from the region of low values of the temperature to the region of high temperatures is effected smoothly. For the low-temperature section of the curves, where $\theta \ll 1$ (physically, this corresponds to the slow flow of a low-viscosity liquid), the dependence of the dissipative heat source on the temperature can be neglected

$$q(\theta) = \omega^2 e^{-\theta} \approx \omega^2 = \text{const} \tag{2.2}$$

With condition (2.2), the solution of Eqs. (1.15) has the form

$$\theta_1 = \Delta\pi^2 B^{-1} (1 - e^{-B/\Delta\pi}), \quad \omega = \Delta\pi \tag{2.3}$$

In the high-temperature region, where $\theta \gg 1$, the adiabatic solution (2.1) is a good approximation.

With certain values of the parameters ($B = B_* \approx 60$, $\Delta\pi = \Delta\pi_* \approx 5.65$), curve 6 rises upwards almost vertically.

With a further increase in the value of B the temperature curves 5, 6, 7 (B = 100, 250, 500) take on an S-shaped form. For the discussion, we select the curve 6, corresponding to the value B = 250. Three branches can be distinguished on the curve: a lower low-temperature branch up to the point ($\Delta\pi_+$, θ_+), corresponding to almost isothermal flow, and intermediate and upper branches beyond the point ($\Delta\pi_-$, θ_-), corresponding to almost adiabatic flow conditions. If the gradually increasing pressure drop $\Delta\pi$ moves along the temperature curve away from the region of low temperatures, then the steady-state heating passes jumpwise from low values to high values. The reverse transition from large degrees of heating to small with a gradual decrease in $\Delta\pi$ also takes place jumpwise. This case is analogous to the phenomenon of ignition and extinction under thermal combustion conditions [8, 9]. Therefore, for the flow under consideration, we can speak of the phenomenon of hydrodynamic ignition and extinction. This phenomenon manifests itself in the fact that, near the first critical point with the coordinates ($\Delta\pi_+$, θ_+), with an infinitely small increase in the pressure drop there is a sharp transition from low-temperature flow conditions to high-temperature conditions, i.e., ignition, while near the second critical point with the coordinates ($\Delta\pi_-$, θ_-) there is the reverse transition, i.e., extinction.

As in the theory of combustion, the critical conditions for ignition and extinction do not coincide. In view of this, we can speak of the hysteresis character of the phenomenon. Hysteresis flow conditions and critical conditions are possible, as can be seen from Fig. 1, with $\Delta\pi > 5.65$. In the case of critical values of the parameters $\Delta\pi_* = 5.65$ and $B_* = 60$, the hysteresis effect vanishes, i.e., the points of ignition and extinction come together into a single point. If $\Delta\pi < 5.65$, then with a change in the heat-transfer coefficient B, the steady-state temperature varies monotonically and we can speak of a process without a crisis.

The phenomenon described above applies to the case where the pressure drop $\Delta\pi$ = const is given as a starting parameter. With a given mass flow rate ω = const, only one set of steady-state conditions is possible and, with a gradual increase in the value of ω, there is a smooth increase in the steady-state heating of the liquid. This can be seen from Fig. 2, which shows the dependence of the relative temperature θ_1/θ_1° (θ_1° is the adiabatic temperature) on $\ln\omega$ (curves 1, 2, 3, 4, 5, 6, 7 correspond to the values B = 1, 10, 30, 60, 100, 250, 500).

The above laws are illustrated on Fig. 3, which shows the dependence $\Delta\pi - \ln\omega$ with different values of the parameter B = 0, 10, 60, 100, 250, 500 (curves 1-6). With any given values of B, a given mass flow

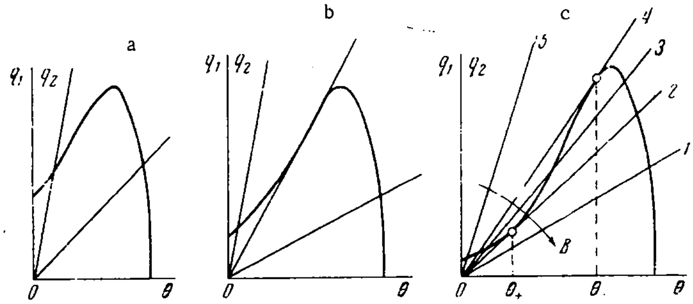

Fig. 4

rate corresponds to a single steady-state value of $\Delta\pi$, while with sufficiently large values of B, a given pressure drop in the interval $\Delta\pi_- < \Delta\pi < \Delta\pi_+$ corresponds to three values of the steady-state mass flow rate: a low value to a small value of the mass flow rate, a high value to a large value of the mass flow rate, while a moderate value represents unstable conditions. With a gradual change in $\Delta\pi$, the transition from small values of the mass flow rate to large values and back is sharp.

3. Let us make an analytical investigation of the above-described phenomenon. For this purpose we use a zero-dimensional method, which has proved fruitful for the description of the qualitative and quantitative aspects of the phenomena of ignition and extinction in the theory of combustion [13], as well as for the description of hysteresis effects for the simplest cases of magnetogasdynamic flows [11, 12]. From a physical point of view, this method corresponds to a transition from a one-dimensional model of ideal displacement to a model of ideal mixing.

In place of the temperature distribution along the length of the tube $\theta(\xi)$, we introduce some mean value of the temperature θ (here and in what follows no special notation is used for the mean temperature). Effecting a finite-difference transition $d\theta/d\xi \sim \theta$, we obtain the following zero-dimensional representation of Eqs. (1.15):

$$\omega\theta = \omega^2 e^{-\theta} - B\theta, \quad \Delta\pi = \omega e^{-\theta} \tag{3.1}$$

For flow conditions with a given pressure drop $\Delta\pi = \text{const}$, eliminating ω from Eq. (3.1), we obtain

$$\Delta\pi\theta e^{\theta} = \Delta\pi^2 e^{\theta} - B\theta \tag{3.2}$$

Denoting

$$q_1(\theta) = \Delta\pi^2 e^{\theta} (1 - \theta / \Delta\pi), \quad q_2(\theta) = B\theta \tag{3.3}$$

Steady-state values of θ correspond to intersection of the curve $q_1(\theta)$ and the straight line $q_2(\theta)$. As can be seen from Fig. 4a, b, c, depending on the value of the parameters $\Delta\pi$ and B, a different character of the intersection of $q_1(\theta)$ and $q_2(\theta)$ is possible: 1) with $\Delta\pi < \Delta\pi_*$ (the value of $\Delta\pi_*$ will be found below), contact between $q_1(\theta)$ and $q_2(\theta)$ is impossible and the solution is unique for any given values of B, and flow conditions without a crisis exist (Fig. 4a); 2) with $\Delta\pi = \Delta\pi_*$, contact between $q_1(\theta)$ and $q_2(\theta)$ is possible at a single point (Fig. 4b); 3) with $\Delta\pi > \Delta\pi_*$, depending on the parameter B (the arrow of Fig. 4c indicates a direction toward the side of a decrease in B), there can be either one point of intersection on the high-temperature branch (straight line 1) and on the low-temperature branch (straight line 5), or three points of intersection (straight line 3).

An investigation of stability analogous to that carried out for the problem of thermal explosion [13] shows that, out of the three possible steady-state sets of conditions, only the high-temperature and the low-temperature are stable while the intermediate is unstable. The case of three points of intersection corresponds to a hysteresis character of the process.

The special qualitative characteristics of the change in the steady-state temperature with a variation of the parameters B and $\Delta\pi$, noted in section 2, are retained also in the simplified zero-dimensional consideration. Let us pass on to a quantitative comparison. Figure 5 gives temperature curves 1 and 2 for B = 30 and B = 100, obtained by calculation using the starting integrodifferential system (1.15), and curves 1' and 2' for these same values of B, obtained by an approximate calculation using a zero-dimensional scheme for the transcendental Eq. (3.2). The corresponding curves obtained by the two methods practically coincide

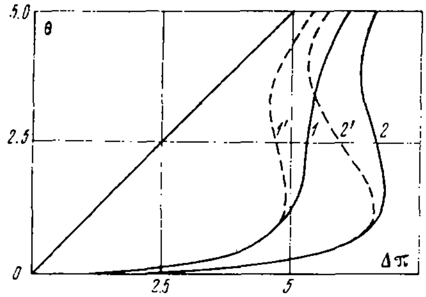

Fig. 5

with small values of $\Delta\pi$ right up to the critical point of ignition. The divergence between them is considerable near the critical point of extinction. An analogous character of the agreement between the calculated characteristics, using the zero-dimensional method, was obtained in [12].

We obtain analytical relationships for finding the critical conditions of hydrodynamic thermal ignition and extinction. It can be seen from Fig. 4c that these conditions correspond to the points of contact between $q_1(\theta)$ and $q_2(\theta)$ (straight lines 2 and 4), i.e., to the points $(\Delta\pi_+, \theta_+)$ and $(\Delta\pi_-, \theta_-)$. To determine them we have the system of equations

$$q_1 = q_2, \quad \Delta\pi^2 e^\theta (1 - \theta / \Delta\pi) = B\theta$$
$$dq_1/d\theta = dq_2/d\theta, \quad \Delta\pi^2 e^\theta (1 - \theta / \Delta\pi - 1 / \Delta\pi) = B \tag{3.4}$$

Substituting B from the first equation into the second, we obtain the quadratic equation

$$\theta^2 - \Delta\pi\theta + \Delta\pi = 0 \tag{3.5}$$

having the following roots:

$$\theta = {}^1/_2\Delta\pi(1 \mp \sqrt{1 - 4/\Delta\pi}) \tag{3.6}$$

Here the minus sign corresponds to $\theta = \theta_+$, i.e., to the critical ignition temperature, and the plus sign to $\theta = \theta_-$, i.e., to the critical extinction temperature. It follows from expression (3.6) that the phenomena of ignition and extinction are possible only with $\Delta\pi > \Delta\pi_* = 4$. Substituting Eq. (3.6) into the second equation of system (3.4), we obtain the critical dependences $B_+ = f_1(\Delta\pi)$ and $B_- = f_2(\Delta\pi)$.

Diagrams of the critical dependences of the heat-transfer coefficients B_+, B_- and the temperatures θ_+, θ_- on the pressure drop $\Delta\pi$ are shown in Fig. 6a, b. The solid lines correspond to an exact solution [in accordance with Eq. (1.15)], and the broken lines to an approximate solution (using the zero-dimensional method). As can be seen from Fig. 6, the whole region is divided into the hysteresis region 1 (in which critical ignition and extinction conditions are possible), and the region without a crisis 2. The hatched region corresponds to the divergence between the exact and approximate solutions. The common point of the curves corresponds to coincidence of the critical ignition and extinction conditions. For this case, from formulas (3.6), (3.4), and (3.1), we have

$$\Delta\pi_* = 4, \quad \theta_* = 2, \quad \omega_* = B_* = 4e^2 \approx 30 \tag{3.7}$$

From the calculations of the steady-state temperature (Fig. 5) and the critical parameters (Fig. 6) it can be seen that the zero-dimensional relationships introduced not only reflect correctly the qualitative side of the phenomenon of hydrodynamic thermal ignition and extinction, but also permit simplified calculational evaluations.

We use relationships (3.7) for a calculation using the example of dimensional critical parameters corresponding to coincidence of the critical ignition and extinction conditions. As a model liquid we take castor oil, for which $c = 0.51$ cal/(g·deg), $\rho = 0.964$ g/cm^3, $k = 0.085$ deg^{-1}, if $T \in (9\text{–}40°C)$ [2]. Let $T_0 = 9°$, R =

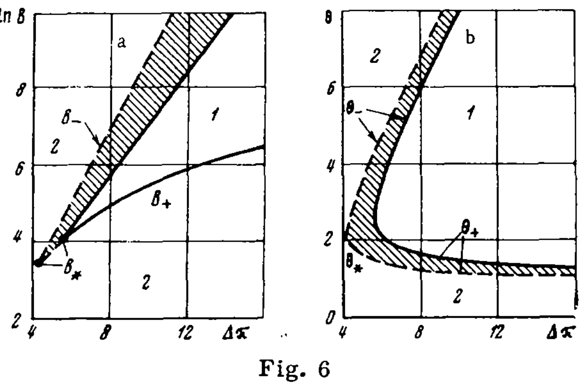

Fig. 6

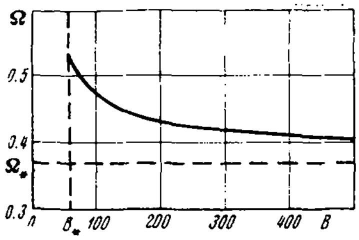

Fig. 7

0.1 cm, l = 20 cm. To these values, from Eq. (3.7), there correspond $\Delta T_* = 23.5°$ C, $\Delta p \approx 1000$ atm, $Q_* = 375$ cm^3/sec, $\alpha_* = 1.47$ cal/(cm^2 · sec · deg), Re = 150. Thus, here there are observed the conditions for laminar flow and for a limited character of the range within which the dependence of the viscosity on the temperature is valid. The critical value of Δp_* can be decreased by preliminary heating of the liquid, so that the inlet temperature T(z = 0) will be greater than the temperature of the surrounding medium T_0.

4. An analysis of the limiting case of an "infinitely long" tube ($B \rightarrow \infty$) is of interest. In this case $\Delta\pi \rightarrow \infty, d\theta(1)/d\xi \rightarrow 0$. The equations describing the process are transformed to the form

$$\Omega e^{\theta} - \theta = 0, \quad \Omega = \Delta\pi^2 / B \tag{4.1}$$

In place of the two parameters $\Delta\pi$ and B determining the thermal flow conditions, for a tube of infinite length there figures the single parameter Ω. Physically, the introduction of Ω means that in place of the pressure drop Δp, the pressure head $b = \Delta\pi/l$ must be considered as a parameter. This was done in [3].

The theory of Eq. (4.1) is known in the theory of thermal explosion of N. N. Semenov [16, 9], from which it follows that

$$\Omega_* = 1 / e \tag{4.2}$$

For a tube of infinite length the phenomenon of extinction vanishes and there are no high-temperature steady-state conditions. In this case, we can speak of the phenomenon of a hydrodynamic thermal explosion [3].

On Fig. 7 the results of calculations of critical ignition conditions for tubes of different length are represented in the form of the dependence Ω_* (B). The divergence between the curve Ω_* (B) and the straight line $\Omega_* = 1/e$ is an effect of the finite dimensions of the tube. This effect is small right up to a value of $B = B_*$ (the greatest divergence is ~30%). The existence of B_* and its value are, in principle, connected with the bounded length of the tube (or with the residence time of the liquid in the tube).

In [3] there is a description of the phenomenon of a hydrodynamic thermal explosion with the flow of a liquid in an infinite cylindrical tube with a thermostatted wall, i.e., in the case where the temperature distribution over the cross section of the tube is considerable. For this case, taking account of the Reynolds dependence of the viscosity on the temperature (1.6), in accordance with [3], the critical conditions for a thermal explosion can be written in the form

$$kb^2R^4/16\lambda\mu(T_0) = 2 \tag{4.3}$$

where λ is the coefficient of thermal conductivity of the liquid; $\mu(T_0)$ is the viscosity with $T = T_0 (\mu(T_0) = \mu_0)$. In the present work an analogous solution to Eq. (4.2) was obtained for the case where there is no temperature distribution over the cross section of the tube; it can be represented in the form

$$kb^2R^3 / 16\alpha\mu_0 = 1 / e \tag{4.4}$$

A comparison between Eqs. (4.3) and (4.4) permits determining the effective heat-transfer coefficient for tubes with a thermostatted wall

$$\alpha = 2e\lambda / R \tag{4.5}$$

This value is twice as great as the analogous value for the case of a chemical thermal explosion [9]. This is connected with the fact that, with a hydrodynamic thermal explosion, the maximum of the rate of heat evolution is located near the surface while with a chemical thermal explosion it is at the center of the tube [3].

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